Archimedean Solids and Operations

WDRP - Polyhedra

Previously, we derived the platonic solids as a special case of polyhedra when all the faces are the same regular polygon. If we expand this to allow combinations of regular polygons we arrive at what are called the Archimedean solids. Instead of deriving them in the same way we did with the platonic solids we will use the platonic solids and performing operations on them to get new shapes. In general, mathematicians are lazy and would rather use something well understood and construct new objects from old ones.

Definition 0.1. An **n-ary operation** is a function $f : S^n \to S$ where S is a set.

Writing S^n just means the operation f has n inputs. It seems like operation and is just another name for function, but it mainly comes down to context. An operation indicates there is some structure on the set S.

Example.

1. A binary operation on the integers $\mathbb{Z}$ is addition $+ : \mathbb{Z}^2 \to \mathbb{Z}$.
2. A unary operation on the integers is additive inverse (not to be confused with subtraction, which is also an operation) $- : \mathbb{Z} \to \mathbb{Z}$.

So what kinds of operations can we perform on polyhedra? There are four we will consider, truncation, rectification, dualizing, and snubification. Truncation is the operation which slices off the corners of the polyhedra, while rectification is pretty much the same, except the cuts don't meet when truncating but do when rectifying. Dualizing is when you put a vertex for each face and a face for each vertex of your polyhedra. Okay so snubification is weird, its like a twist combined with adding triangles. I'm not going to try to describe it cause it's not gonna turn out good, instead you should look at all of these operations in polyhedra viewer.

Theres a couple of things to point out, for one, each of these operations is a unary operation, only one input. Also, the platonic solids are closed under dualizing, that is, each platonic solid is the dual of another platonic solids. In particular the tetrahedron is dual to itself, the cube and octahedron are dual and the dodecahedron and icosahedron are dual. The table below lists all 13 Archimedean solids and how they are constructed either from the platonic solids or each other. Some of the names are pretty self explanatory though.

Truncated cube	Truncate the Cube
Truncated tetrahedron	Truncate the Tetrahedron
Truncated Octahedron	Truncate the Octahedron
Truncated Dodecahedron	Truncate the Dodecahedron
Truncated Icosahedron (Soccer ball)	Truncate the Icosahedron
Cuboctrahedron	Rectify the Cube or Octahedron
Icosidodecahedron	Rectify the Isocahedron or Dodecahedron
Truncated Icosidodecahedron (Great Rhombicosidodecahedron)	Truncate the Icosidodecahedron
Truncated Cuboctrahedron (Great Rhombicuboctahedron)	Truncate the Cuboctahedron
(Small) Rhombicosidodecahedron	Rectify the Icosidodecahedron
(Small) Rhombicuboctahedron	Rectify the Cuboctahedron
Snub Cube	Snubify the Cube or Octahedron
Snub Dodecahedron	Snubify the Dodecahedron or Icosahedron